Math 72 : 6.1 Simplifying Rational Expressions

\+ 6.2 Multiplying + Dividing Rational Expressions.

Objectives

1) Evaluate a rational expression
2) Find the domain of a rational expression
3) Simplify a rational expression
4) Multiply rational expressions
5) Divide rational expressions

Rational ("ratio" at its core) means fractional

Simplifying Rational Expressions

Objectives:

1) Evaluate a rational expression at a given value
   a. Substitute the given value for all locations of that variable
   b. Use parentheses if substituting a negative number
   c. Final answer is a number, or "undefined"
2) Determine the values of the variable that make the value of the rational expression undefined
   a. "Undefined" happens when evaluating causes divide by zero.
   b. Only the denominator causes divide by zero.
   c. Set only the denominator equal to zero, and solve
3) Simplify rational expressions
   a. Factor completely
   b. Divide out common factors

Practice:

1) Evaluate $\dfrac{p^2-9}{2p^2+p-10}$ for the following values of p
   a. $p=-1$
   b. $p=0$
   c. $p=2$
   d. $p=-\dfrac{5}{2}$

2) a) Find the values of the variable that make the expression $\dfrac{3h+2}{h^3+5h^2+4h}$ undefined.
b) and write the domain.

Simplify the rational expression.

3) $\dfrac{4-x^2}{2x^2-x-6}$

4) $\dfrac{ab+3b-ac-3c}{a^2+6a+9}$

5) $\dfrac{x-3}{x^3-27}$

6) $\dfrac{x^3+64}{x^2-4x+16}$

7) $\dfrac{4p^2-20pq+25q^2}{6p^2-7pq-20q^2}$

8) $\dfrac{3x^3+3x^2-36x}{6x^3-6x^2-120x}$

9) $\dfrac{x^3+4x}{x^4-16}$

Simplifying Rational Expressions

Objectives: 1) Evaluate a rational expression
2) Determine values of variable that make value of the rational expression undefined. + write the domain
3) Simplify rational expression
- factor completely
- divide out common factors.

Defn: A rational expression is a quotient of two polynomials $\frac{p}{q}$ where $q \neq 0$.

Generally speaking: a rational expression is a fraction with in variable in the denominator.

① Evaluate $\frac{p^2-9}{2p^2+p-10}$ for a) $p=-1$ b) $p=0$ c) $p=2$ d) $p=-5/2$

step 1: Substitute (using ( ) for negatives)

step 2: Use order of operations to calculate.

a) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{(-1)^2-9}{2(-1)^2+(-1)-10}$

$= \frac{1-9}{2(1)-1-10}$

$= \frac{-8}{2-1-10}$

$= \frac{-8}{-9}$

$= \boxed{\frac{8}{9}}$

b) $\frac{p^2-9}{2p^2+p-10} \rightarrow \frac{0^2-9}{2(0)^2+0-10}$

$= \frac{-9}{-10}$

$= \boxed{\frac{9}{10}}$

① con't

c) $\dfrac{p^2-9}{2p^2+p-10} \rightarrow \dfrac{2^2-9}{2(2)^2+2-10}$

$= \dfrac{4-9}{2\cdot 4+2-10}$

$= \dfrac{-5}{8+2-10}$

$= \dfrac{-5}{10-10}$

$= \dfrac{-5}{0}$

$= \boxed{\text{undefined}}$

d) $\dfrac{p^2-9}{2p^2+p-10} \rightarrow \dfrac{(-5/2)^2-9}{2(-5/2)^2+(-5/2)-10}$

$= \dfrac{\frac{25}{4}-9}{2\left(\frac{25}{4}\right)-\frac{5}{2}-10}$

$= \dfrac{\frac{25}{4}-\frac{9}{1}\cdot\frac{4}{4}}{\frac{50}{4}-\frac{5}{2}\cdot\frac{2}{2}-\frac{10}{1}\cdot\frac{4}{4}}$

$= \dfrac{\left(\frac{25-36}{4}\right)}{\left(\frac{50-10-40}{4}\right)}$

$= \dfrac{\frac{-9}{4}}{\left(\frac{0}{4}\right)}$

$= \left(-\frac{9}{4}\right) \div \left(\frac{0}{4}\right)$

$= -\frac{9}{4} \div 0$

$= \boxed{\text{undefined}}$

Review: Solve $2p^2 + p - 10 = 0$

-20; 5, -4; 1

$$2p^2 + 5p - 4p - 10 = 0$$

$$p(2p+5) - 2(2p+5) = 0$$

$$(2p+5)(p-2) = 0$$

$2p + 5 = 0$ $\qquad$ $p - 2 = 0$

$2p = -5$ $\qquad$ $\boxed{p = 2}$

$$\boxed{p = \frac{-5}{2}}$$

NOTICE: 1) $2p^2 + p - 10$ is the denominator of $\dfrac{p^2-9}{2p^2+p-10}$ in ①.

2) The solutions of $2p^2 + p - 10 = 0$ where $-\frac{5}{2}$ and 2.

3) $\frac{-5}{2}$ and 2 were the numbers in ① c) and ① d) that gave undefined results.

4) Undefined means $\div 0$, or denominator $= 0$.

5) To find values of variable that make a rational expression undefined, we need denominator $= 0$.

The domain is the set of all values of x which result in real and defined y-values.

② Find the values for which $\dfrac{3h+2}{h^3+5h^2+4h}$ is undefined.

a) $h^3 + 5h^2 + 4h = 0$

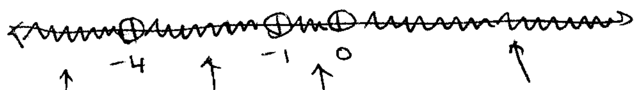
$h(h^2 + 5h + 4) = 0$

$h(h+4)(h+1) = 0$

$\boxed{h = 0, -4, -1}$

b) The domain is all values of h which are real numbers except $0, -4, -1$

$\boxed{\{h : h \neq 0, -4, -1\}}$ or $\boxed{\{h : h \in \mathbb{R},\ h \neq 0, -4, -1\}}$ set notation



$\boxed{(-\infty, -4) \cup (-4, -1) \cup (-1, 0) \cup (0, \infty)}$ interval notation

To simplify a rational expression

Step 0: Write numerator and denominator in standard form. * This is Important. *

Step 1: Factor everything completely.
* This is essential. There is no skipping this step, no matter how close you come. *

Step 2: Identify common factors in the numerator and divide (cancel) them out.

Step 3: Leave final answer fully factored.
(Don't FOIL or multiply.)

(3) $\dfrac{4-x^2}{2x^2-x-6}$

$= \dfrac{-x^2+4}{2x^2-x-6}$

$= \dfrac{-(x^2-4)}{2x^2-x-6}$

$= \dfrac{-(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(2x+3)}$

$= \boxed{\dfrac{-(x+2)}{(2x+3)}}$

-12
-4 3
-1

denominator
$2x^2-x-6$
$2x^2-4x+3x-6$
$2x(x-2)+3(x-2)$
$(x-2)(2x+3)$

(4) $\dfrac{ab+3b-ac-3c}{a^2+6a+9}$

$= \dfrac{(a+3)(b-c)}{(a+3)(a+3)}$

$= \boxed{\dfrac{b-c}{a+3}}$

numerator grouping:
$ab+3b-ac-3c$
$b(a+3)-c(a+3)$
$(a+3)(b-c)$

perfect square trinomial for denominator

Simplify.

(5) $\dfrac{x-3}{x^3-27}$

diff of cubes

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$x^3-27=(x-3)(x^2+3x+9)$

$= \dfrac{\cancel{(x-3)}}{\cancel{(x-3)}(x^2+3x+9)}$

$= \boxed{\dfrac{1}{x^2+3x+9}}$

(6) $\dfrac{x^3+64}{x^2-4x+16}$

sum of cubes

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$x^3+64=(x+4)(x^2-4x+16)$

Notice! the trinomial is the same as the denominator!

$= \dfrac{(x+4)\cancel{(x^2-4x+16)}}{\cancel{(x^2-4x+16)}}$

$= \boxed{x+4}$

(7) $\dfrac{4p^2-20pq+25q^2}{6p^2-7pq-20q^2}$

numerator

perfect square trinomial

$(2p-5q)(2p-5q)$

$4p^2-10pq-10pq+25q^2$ ✓

denominator

$6p^2-7pq-20q^2$

−120

−15 8

−7

1, −12
2, −6
3, −4
4, −3
5, −2
6, −2
8, −15

$=6p^2-15pq+8pq-20q^2$

$=3p(2p-5q)+4q(2p-5q)$

$=(2p-5q)(3p+4q)$

$= \dfrac{\cancel{(2p-5q)}(2p-5q)}{\cancel{(2p-5q)}(3p+4q)}$

$= \boxed{\dfrac{2p-5q}{3p+4q}}$

(8) $\dfrac{3x^3+3x^2-36x}{6x^3-6x^2-120x}$

$= \dfrac{3x\cancel{(x+4)}(x-3)}{6x\cancel{(x+4)}(x-5)}$

$= \boxed{\dfrac{(x-3)}{2(x-5)}}$

$\dfrac{\cancel{x}}{\cancel{x}} = 1$

$\dfrac{3}{6} = \dfrac{1}{2}$

numerator

$3x^3+3x^2-36x$

$3x(x^2+x-12)$

$3x(x+4)(x-3)$

GCF

-12; 4, -3; 1

denom

$6x^3-6x^2-120x$

$6x(x^2-x-20)$

$6x(x-5)(x+4)$

-20; -5, $+4$; -1

(9) $\dfrac{x^3+4x}{x^4-16}$

$= \dfrac{x\cancel{(x^2+4)}}{(x-2)(x+2)\cancel{(x^2+4)}}$

$= \boxed{\dfrac{x}{(x-2)(x+2)}}$

numerator

x^3+4x

$x(x^2+4)$

sum of squares is prime.

denominator

x^4-16

difference of squares

$= (x^2-4)(x^2+4)$

another diff of sq.

$= (x-2)(x+2)(x^2+4)$

Multiplying and Dividing Rational Expressions

Objectives:

1) Multiply two rational expressions
   a. Factor everything completely
   b. Divide out common factors
2) Divide two rational expressions
   a. Rewrite as multiply by reciprocal of the second fraction
   b. Factor everything completely
   c. Divide out common factors

Practice:

Perform the indicated operations and simplify.

✓ 1) $\frac{p^2-9}{p^2+5p+6}\cdot\frac{p+2}{6-2p}$

✓ 2) $\frac{m^2-n^2}{10m^2-10mn}\cdot\frac{10m+5n}{2m^2+3mn+n^2}$

✓ 3) $\frac{4-z^2}{3z-2}\cdot\frac{3z^2+7z-6}{z^2+z-6}$

4) $\frac{n^2-n-2}{4n^2-9}\cdot\frac{2n^2-n-6}{2n^2-5n+3}$

5) $\frac{8n-8}{n^2-3n+2}\cdot\frac{2n^2+9n+10}{12}$

✓ 6) $\frac{y^2-9}{2y^2-y-15}\div\frac{3y^2+10y+3}{2y^2+y-10}$

✓ 7) $\frac{a^2+3a+2}{a^2-9}\div(a+2)$

8) $\dfrac{\frac{12x}{5x+20}}{\frac{4x^2}{x^2-16}}$

9) $\dfrac{\frac{x^2+25}{x^7}}{\frac{x^2+5x}{x^2}}$

✓ 10) $\frac{x^2-5x}{x^3+125}\div\frac{x-5}{x^2-5x+25}$

① $\dfrac{p^2-9}{p^2+5p+6} \cdot \dfrac{p+2}{6-2p}$

$= \dfrac{\cancel{(p+3)}\cancel{(p-3)}}{\cancel{(p+2)}\cancel{(p+3)}} \cdot \dfrac{\cancel{(p+2)}}{(-2)\cancel{(p-3)}}$

$= \dfrac{1}{-2}$

$= \boxed{\dfrac{-1}{2}}$

Factor completely

1) $p^2-9 = (p-3)(p+3)$

2) p^2+5p+6 6, 2, 3, 5 $= (p+2)(p+3)$

3) $p+2$ already factored $(p+2)$

4) $6-2p = -2p+6$ standard form

$= -2(p-3)$ GCF w/ neg.

Cancel common factors

$\dfrac{p+3}{p+3} \cdot \dfrac{p-3}{p-3} \cdot \dfrac{p+2}{p+2}$

② $\dfrac{m^2-n^2}{10m^2-10mn} \cdot \dfrac{10m+5n}{2m^2+3mn+n^2}$

$= \dfrac{\cancel{(m-n)}\cancel{(m+n)}}{10m\cancel{(m-n)}} \cdot \dfrac{5\cancel{(2m+n)}}{\cancel{(m+n)}\cancel{(2m+n)}}$

$= \dfrac{5}{10m}$

$= \boxed{\dfrac{1}{2m}}$

Factor completely.

1) $m^2-n^2 = (m-n)(m+n)$

difference of two squares

2) $10m^2-10mn = 10m(m-n)$

GCF $10m$

3) $10m+5n = 5(2m+n)$

GCF$=5$

4) $2m^2+3mn+n^2$ $2m^2n^2$, $1mn$, $2mn$, $3mn$

$= 2m^2+2mn+1mn+n^2$

$= 2m(m+n)+n(m+n)$

$= (m+n)(2m+n)$

③ $\dfrac{4-z^2}{3z-2} \cdot \dfrac{3z^2+7z-6}{z^2+z-6}$

$= \dfrac{-\cancel{(z-2)}(z+2)}{\cancel{(3z-2)}} \cdot \dfrac{\cancel{(z+3)}\cancel{(3z-2)}}{\cancel{(z+3)}\cancel{(z-2)}}$

$= -(z+2)$

$= \boxed{-z-2}$

Final answer is <u>not</u> a fraction. Simplify by distribute.

If final answer is a fraction, leave factored

Factor completely.

1) $4-z^2$

$= -z^2+4$ standard form

$= -(z^2-4)$ GCF $= -1$

$= -(z-2)(z+2)$ diff of two squares

⇑ keep the (-1) in the answer.

2) $3z-2$ already factored

$= (3z-2)$

3) $3z^2+7z-6$ $\quad -18 \quad 9 \quad -2 \quad 7$

$= 3z^2+9z-2z-6$

$= 3z(z+3)-2(z+3)$

$= (z+3)(3z-2)$

4) z^2+z-6 $\quad -6 \quad 3 \quad -2 \quad 1$

$(z+3)(z-2)$

④ $\dfrac{n^2-n-2}{4n^2-9} \cdot \dfrac{2n^2-n-6}{2n^2-5n+3}$

$= \dfrac{(n-2)(n+1)}{(2n-3)\cancel{(2n+3)}} \cdot \dfrac{(n-2)\cancel{(2n+3)}}{(2n-3)(n-1)}$

$= \dfrac{(n-2)(n+1)(n-2)}{(2n-3)(2n-3)(n-1)}$

$= \boxed{\dfrac{(n+1)(n-2)^2}{(n-1)(2n-3)^2}}$

Leave answer factored

<u>Note</u>: If the similar factors are both in the numerator (or both in the denominator) they <u>cannot</u> be canceled.

1) n^2-n-2 $\quad -2 \quad -2 \quad 1 \quad -1$

$(n-2)(n+1)$

2) $4n^2-9$ diff of sq.

$= (2n-3)(2n+3)$

3) $2n^2-n-6$ $\quad -12 \quad -4 \quad 3 \quad -1$

$= 2n^2-4n+3n-6$

$= 2n(n-2)+3(n-2)$

$= (n-2)(2n+3)$

4) $2n^2-5n+3$ $\quad 6 \quad -3 \quad -2 \quad -5$

$= 2n^2-3n-2n+3$

$= n(2n-3)-1(2n-3)$

$= (2n-3)(n-1)$

5. $$\frac{8n-8}{n^2-3n+2} \cdot \frac{2n^2+9n+10}{12}$$

$$= \frac{8\cancel{(n-1)}}{(n-2)\cancel{(n-1)}} \cdot \frac{(n+2)(2n+5)}{12}$$

$$= \frac{8}{12} \cdot \frac{(n+2)(2n+5)}{(n-2)}$$

$$= \boxed{\frac{2(n+2)(2n+5)}{3(n-2)}}$$

Note: $(n+2)$ and $(n-2)$ are two different numbers.

For example: If $n=3$

$n+2 \Rightarrow 3+2=5$

$n-2 \Rightarrow 3-2=1$

1) $8n-8 = 8(n-1)$ GCF $= 8$

2) n^2-3n+2

$(n-2)(n-1)$

2, -2, -1, -3

3) $2n^2+9n+10$

20, 4, 5, 9

$= 2n^2+4n+5n+10$

$= 2n(n+2)+5(n+2)$

$= (n+2)(2n+5)$

4) 12

multiply by reciprocal

6. $$\frac{y^2-9}{2y^2-y-15} \div \frac{3y^2+10y+3}{2y^2+y-10}$$

$$= \frac{y^2-9}{2y^2-y-15} \cdot \frac{2y^2+y-10}{3y^2+10y+3}$$

$$= \frac{\cancel{(y-3)}\cancel{(y+3)}}{\cancel{(y-3)}\cancel{(2y+5)}} \cdot \frac{\cancel{(2y+5)}(y-2)}{\cancel{(y+3)}(3y+1)}$$

$$= \boxed{\frac{y-2}{3y+1}}$$

factor completely

1) $y^2-9 = (y-3)(y+3)$

difference of squares

2) $2y^2-y-15$

-30, -6, $+5$, -1

$= 2y^2-6y+5y-15$

$= 2y(y-3)+5(y-3)$

$= (y-3)(2y+5)$

3) $2y^2+y-10$

-20, 5, -4, 1

$= 2y^2+5y-4y-10$

$= y(2y+5)-2(2y+5)$

$= (2y+5)(y-2)$

4) $3y^2+10y+3$

9, 9, 1, 10

$= 3y^2+9y+y+3$

$= 3y(y+3)+1(y+3)$

$= (y+3)(3y+1)$

⑦ $\dfrac{a^2+3a+2}{a^2-9} \div (a+2)$

$= \dfrac{a^2+3a+2}{a^2-9} \cdot \dfrac{1}{(a+2)}$ multiply by reciprocal

factor completely

1) $a^2+3a+2 = (a+2)(a+1)$

2) $a^2-9 = (a-3)(a+3)$

3) 1

4) $(a+2)$

$= \dfrac{\cancel{(a+2)}(a+1)}{(a+3)(a-3)} \cdot \dfrac{1}{\cancel{(a+2)}}$

$= \boxed{\dfrac{(a+1)}{(a+3)(a-3)}}$

Recall: Leave fractions factored.

⑧ $\dfrac{\;\dfrac{12x}{5x+20}\;}{\;\dfrac{4x^2}{x^2-16}\;}$ ← This division bar becomes ÷ symbol.

$= \dfrac{\left(\dfrac{12x}{5x+20}\right)}{\left(\dfrac{4x^2}{x^2-16}\right)}$

$= \dfrac{12x}{5x+20} \div \dfrac{4x^2}{x^2-16}$ write with ÷ symbol

$= \dfrac{12x}{5x+20} \cdot \dfrac{x^2-16}{4x^2}$ multiply by reciprocal

$= \dfrac{12 \cdot x}{5\cancel{(x+4)}} \cdot \dfrac{(x-4)\cancel{(x+4)}}{4x^2}$ factor completely

$= \dfrac{12 \cdot x \cdot (x-4)}{4 \cdot 5 \cdot x^2}$ reduce $\frac{12}{4} = 3$, $\frac{x}{x^2} = \frac{1}{x}$

$= \boxed{\dfrac{3(x-4)}{5x}}$ leave result factored

⑨ $$\frac{\frac{x^2+25}{x^7}}{\frac{x^2+5x}{x^2}}$$

$$= \frac{x^2+25}{x^7} \div \frac{x^2+5x}{x^2}$$ rewrite as division

$$= \frac{(x^2+25)}{x^7} \cdot \frac{x^2}{x^2+5x}$$ rewrite as multiply by reciprocal of 2nd fraction

$$= \frac{(x^2+25)}{x^7} \cdot \frac{x^2}{x(x+5)}$$ sum of squares x^2+25 is prime
factor GCF x from x^2+5x

$$= \frac{x^2}{x^8} \cdot \frac{(x^2+25)}{(x+5)}$$ rewrite like bases near each other

$$= \boxed{\frac{(x^2+25)}{x^6(x+5)}}$$ exponent law

⑩ $$\frac{x^2-5x}{x^3+125} \div \frac{x-5}{x^2-5x+25}$$

$$= \frac{x^2-5x}{x^3+125} \cdot \frac{x^2-5x+25}{x-5}$$ rewrite as multiply by reciprocal of 2nd fraction

$$= \frac{x(x-5)}{(x+5)(x^2-5x+25)} \cdot \frac{(x^2-5x+25)}{(x-5)}$$ factor GCF x from x^2-5x
factor sum of cubes x^3+125
add parentheses to prime polynomials $(x^2-5x+25)$ and $(x-5)$

$$= \frac{x\,\cancel{(x-5)}\,\cancel{(x^2-5x+25)}}{\cancel{(x-5)}\,\cancel{(x^2-5x+25)}\,(x+5)}$$ write like bases near each other & cancel common factors

$$= \boxed{\frac{x}{x+5}}$$